Eur. Phys. J. C 45, 693–700 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2005-02432-5

THE EUROPEAN
PHYSICAL JOURNAL C

Annihilation contribution and $B \to a_0\pi$, f_0K decays

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Received: 17 May 2005 /
Published online: 24 January 2006 –

Abstract. We analyze the $B^0 \to a_0^{\pm}\pi^{\mp}$ and $B^{-,0} \to f_0K^{-,0}$ decays and show that a consistent phenomenological picture can be obtained within the factorization approximation. In this approach the O_6 operator provides the dominant contributions to the suppressed channel $B^0 \to a_0^+\pi^-$. An estimate of the annihilation form factor using perturbative QCD indicates that this contribution is not negligible, moreover interference with other penguin contributions includes a free parameter since the phase of the annihilation amplitude is not determined. Assumptions based on $SU(2)$ isospin symmetry provide relations between different B decays involving one scalar and one pseudoscalar meson.

1 Introduction

B factories provide large samples of B–$\bar{B}$ mesons allowing for the study of physical phenomena such as CP-violation, the determination of the CKM mixing angles and the search for new physics [1,2]. Clearly, hadronic physics will benefit for the high statistics achieved, and the study of processes with small branching ratios will be possible. Full understanding of B physics is still lacking, as well as a systematic, first-principles description of the phenomena involved. Instead, diverse theoretical approaches are compared to data and assumptions such as factorization, or the estimation of the relative size of different contributions (tree level, annihilation, penguins, final state interactions) can be tested. This can be achieved in processes where the dominant contributions are suppressed by symmetry or accidental cancelations.

The BABAR and Belle collaborations already reported precise measurements of non-leptonic B meson decays involving scalar mesons with a branching ratio of the order of as low as 10^{-6}. Thus for example, for the $B^0 \to f_0K^0$ channel, besides the branching ratio the CP-violating asymmetries are reported and, from the two pion spectrum, the authors are able to obtain the mass and width of the $f_0(980)$ [3]. This is not the case for $B \to a_0(980)\pi$ where the branching fraction for given final states are reported – in particular $a_0^-\pi^+$ – however, in this case it is not possible to separate B^0 from $\bar{B}^0$ decays, unless a dominant decay mechanism is assumed [4]. In this context it is worth remarking that the $B \to a_0(980)\pi$ decay was suggested as a place where α, the weak mixing angle, could be measured through the CP-violating asymmetries [5]. However, it was shown that $B \to a_0^+\pi^-$ is suppressed by G parity and also by isospin, which implies that in the symmetry limit no CP-violating asymmetry is expected to be experimentally accessible [6]. Thus, theoretical arguments support the idea that $B^0 \to a_0^+\pi^-$ is strongly suppressed, so that the reported branching ratio can be identified with the dominant $B^0 \to a_0^-\pi^+$ decay.

The low lying scalar sector of QCD represents a major challenge [7]. From the experimental point of view, the nature of the existing states has not been elucidated while from the theory side no consistent interpretation of the experimental data exists [8]. This is so even though a number of processes involving the appropriated final state in the kinematical region of interest have been analyzed. Thus, for example, $\phi \to \pi\pi\gamma$, $J/\Psi \to \phi\pi\pi$, ϕKK and central production involve the $f_0(980)$ and $a_0(980)$ [9], whereas the di-pion in the $\Upsilon(nS) \to \Upsilon(mS)\pi\pi$ and $D \to \pi\pi\pi$ decays include the kinematical region where the $f_0(600)$ is expected to appear [10–12]. Unfortunately, although different processes are included in the analysis, data are not good enough to provide a clear picture of the scalars. In fact no consensus exists even on the fundamental intrinsic properties (mass and width) of the low lying scalar mesons [2,11].

The appropriate theoretical description of non-leptonic B decays involving scalars is important not only to understand the nature of the scalar mesons but also because they represent a background to other processes of interest in B physics. Since scalars, vectors and tensors couple to two pseudoscalars, the following decays lead to the same final state: $B \to PV, B \to SP, B \to TP$ and $B \to PPP$, where S, V, T and P stand for scalar, vector, spin 2 and pseudoscalar meson respectively. B decays involving scalar

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mesons have been considered by a number of authors. Thus for example in [13] the tree level Hamiltonian and quark model calculations are used to predict branching ratios, while sum rules [14] and gluon-penguin dominance ($b \to sg$) are the basis to interpret the scalars produced in B decays in terms of glue balls [15], or, again, using the QCD corrected Hamiltonian plus factorization the authors in [16] propose evidence for the two quark nature of the $f_0(980)$.

In this work we analyze the $B^0 \to f_0(980)K$ and $B \to a_0(980)\pi$ decays using the factorization approximation. To this end we use the $\Delta B = 1$ weak Hamiltonian including QCD corrections to next to leading order. To evaluate the matrix elements we use values reported in the literature, or model-dependent estimates of the decay constants and form factors. In particular, the annihilation contribution is evaluated using perturbative QCD which is important in estimating contributions previously neglected.

2 Amplitudes for $\bar{B}^0 \to a_0^{\pm}\pi^{\mp}$ and $B^{0,-} \to f_0K^{0,-}$

Following the conventional approach [17–21], we start with the $\Delta B = 1$ effective Hamiltonian $H_{\text{eff}}(q = d, s)$,

$$\begin{aligned}
&\mathcal{H}_{\text{eff}} \\
&= \frac{G_{\text{F}}}{\sqrt{2}}\left[\lambda_{uq}(C_1O_1^u + C_2O_2^u) - \lambda_{tq}\left(\sum_{i=3}^{10} C_iO_i + C_gO_g\right)\right] \\
&\quad + \text{h.c.},
\end{aligned} \tag{1}$$

where $\lambda_{q'q} = V_{q'b}V^*_{q'q}, q = d, s, q' = u, c, t, V_{ij}$ are the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. The Wilson coefficients C_i, including next to leading order QCD corrections, are evaluated at the renormalization scale $\mu \simeq m_B/2$. We use the conventions and values for the C_i constants reported in [19]. With these elements at hand it remains to evaluate the matrix elements of the effective Hamiltonian between the states of interest:

$$A(B \to PS) = \langle PS|H_{\text{eff}}|B\rangle; \tag{2}$$

P and S stand for pseudoscalar and scalar meson respectively. In terms of the amplitude the branching ratio is given by

$$\text{Br}(B \to PS) \simeq \tau_B \frac{G_{\text{F}}^2|A(B \to PS)|^2}{32\pi m_B}, \tag{3}$$

with τ_B the appropriate B meson lifetime ($\tau_{B^+} = 1.65 \cdot 10^{-12}\,\text{s}, \tau_{B^0} = 1.56 \cdot 10^{-12}\,\text{s}$). The matrix elements are evaluated using factorization, i.e. by inserting the vacuum between the currents in all possible ways, and are given by

$$\begin{aligned}
&A_{\bar{B}^0 \to \pi^- a_0^+} \\
&\simeq \lambda_{ud}\left(a_1 X^{\pi^-}_{\bar{B}^0 a_0^+} + a_2 X^{\bar{B}^0}_{(a_0^+\pi^-)_u}\right) \\
&\quad -\lambda_{td}\Bigg[\left(a_4 + a_{10} - \frac{(a_6 + a_8)m_\pi^2}{\widehat{m}(m_b + m_u)}\right) X^{\pi^-}_{\bar{B}^0 a_0^+} \\
&\quad + \left(2(a_3 - a_5) + a_4 + \frac{a_9 - a_7 - a_{10}}{2}\right. \\
&\quad \left. - \frac{(a_6 - a_8/2)m_B^2}{m_u(m_b + m_d)}\right) X^{\bar{B}^0}_{(a_0^+\pi^-)_u}\Bigg],
\end{aligned} \tag{4}$$

$$\begin{aligned}
&A_{\bar{B}^0 \to \pi^+ a_0^-} \\
&\simeq \lambda_{ud}\left(a_1 X^{a_0^-}_{\bar{B}^0 \pi^+} + a_2 X^{\bar{B}^0}_{(a_0^-\pi^+)_u}\right) \\
&\quad -\lambda_{td}\Bigg[(a_4 + a_{10}) X^{a_0^-}_{\bar{B}^0\pi^+} - 2(a_6 + a_8)\tilde{X}^{a_0^-}_{\bar{B}^0\pi^+} \\
&\quad + \left(2(a_3 - a_5) + a_4 + \frac{a_9 - a_7 - a_{10}}{2}\right. \\
&\quad \left. - \frac{(a_6 - a_8/2)m_B^2}{m_u(m_d + m_b)}\right) X^{\bar{B}^0}_{(a_0^-\pi^+)_u}\Bigg],
\end{aligned} \tag{5}$$

$$\begin{aligned}
&A_{B^- \to \pi^0 a_0^-} \\
&\simeq \lambda_{ud}\left(a_1\left(X^{a_0^-}_{B^-\pi^0} + X^{B^-}_{a_0^-\pi^0}\right) + a_2 X^{\pi^0_u}_{B^- a_0^-}\right) \\
&\quad -\lambda_{td}\Bigg[(a_4 + a_{10}) X^{a_0^-}_{B^-\pi^0} - 2(a_6 + a_8)\tilde{X}^{a_0^-}_{B^-\pi^0} \\
&\quad - \left(a_4 - \frac{3}{2}(a_9 - a_7) - \frac{1}{2}a_{10} + \frac{(a_6 + a_8)m_\pi^2}{m_u(m_b + m_d)}\right) X^{\pi^0_u}_{B^- a_0^-} \\
&\quad + \left(a_4 + a_{10} + \frac{(a_6 + a_8)m_B^2}{\widehat{m}(m_b + m_u)} X^{B^-}_{a_0^-\pi^0}\right)\Bigg],
\end{aligned} \tag{6}$$

$$\begin{aligned}
&A_{B^- \to \pi^- a_0^0} \\
&\simeq \lambda_{ud} a_1\left(X^{\pi^-}_{B^- a_0^0} + X^{B^-}_{a_0^0\pi^-}\right) \\
&\quad -\lambda_{td}\Bigg[\left(a_4 + a_{10} - \frac{(a_6 + a_8)m_\pi^2}{\widehat{m}(m_b + m_u)}\right) X^{\pi^-}_{B^- a_0^0} \\
&\quad + \left(a_4 + a_{10} - \frac{(a_6 + a_8)m_B^2}{\widehat{m}(m_b + \widehat{m})}\right) X^{B^-}_{a_0^0\pi^-} \\
&\quad + (a_8 - 2a_6)\tilde{X}^{a_d^0}_{B^-\pi^-}\Bigg],
\end{aligned} \tag{7}$$

$$\begin{aligned}
&A_{B^- \to f^0 K^-} \\
&\simeq \lambda_{us} a_1 \left[X^{K^-}_{B^- f^0} + X^{B^-}_{f^0 K^-}\right] \\
&\quad -\lambda_{ts}\Bigg[\left(a_4 + a_{10} - \frac{2(a_6 + a_8)m_B^2}{(m_b + m_u)(m_s + m_u)}\right) X^{B^-}_{f^0 K^-} \\
&\quad + \left(a_4 + a_{10} - \frac{2(a_6 + a_8)m_K^2}{(m_u + m_s)(m_b + m_u)}\right) X^{K^-}_{B^- f^0} \\
&\quad - (2a_6 + a_8)\tilde{X}^{f_s^0}_{B^- K^-}\Bigg],
\end{aligned} \tag{8}$$

$$\begin{aligned}
&A_{\bar{B}^0 \to f^0 \bar{K}^0} \\
&\simeq -\lambda_{ts}\Bigg[\left(a_4 - \frac{a_{10}}{2} - \frac{(2a_6 - a_8)m_B^2}{(m_b + m_d)(m_s + m_d)}\right) X^{\bar{B}^0}_{f^0 \bar{K}^0}
\end{aligned}$$

Table 1. Numerical values for the effective coefficients a_i^{eff} for $b \to d$ transitions at scale $\mu \approx m_b$ (for $a_3, \ldots, a_{10}$ in units of 10^{-4} ($a_{2i-1} = C_{2i-1} + C_{2i}/N$, $a_{2i} = C_{2i} + C_{2i-1}/N$))

Refs.	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}
[18]	1.039	0.084	40	−440	−120	−620	0.7 − i	4.7 − 0.3i	−94 − i	−14 − 0.3i
[17]	1.050	0.053	48	−412 − 36i	−45	−548 − 36i	0.7 − i	4.7 − 0.3i	−94 − i	−14 − 0.3i
[19]	1.046	0.024	72 − 0.3i	−379 − 102i	−27 − 0.3i	−431 − 102i	−0.81 − 2.4i	3.3 − 0.8i	−92.4 − 2.41i	0.34 − 0.8i
[20]	1.061	0.011	63	−317	−60	−473	4	5.4	−87	−2.4
[21]	1.029	0.103	36	−228	−24	−298	12	7.6	−82	−8.2

$$+\left(a_4 - \frac{a_{10}}{2} - \frac{(2a_6 - a_8)m_K^2}{(m_s + m_d)(m_b + m_d)}\right) X^{\bar{K}^0}_{\bar{B}^0 f^0}$$
$$-(2a_6 - a_8)\tilde{X}^{f_s^0}_{\bar{B}^0 \bar{K}^0}\Bigg], \tag{9}$$

where $\widehat{m} = (m_u + m_d)/2$. For future reference we quote in Table 1 the numerical values of the a_i coefficients. The $X^a_{b,c}$ reduce to products of matrix elements of a current, each of which can be parameterized in terms of form factors or decay constants. Below we present a typical example, and we leave the detailed definitions of the $X^a_{b,c}$ for the appendix. We have

$$X^{\bar{B}^o}_{\bar{K}^0 f_0} = \langle \bar{K}^0 f^0 | (\bar{s}d)_{\text{L}} | 0 \rangle \langle 0 | (\bar{d}b)_{\text{L}} | \bar{B}^0 \rangle$$
$$= f_B (m_{f^0}^2 - m_K^2) F_0^{f^0 \bar{K}^0}(m_B^2). \tag{10}$$

Let us summarize our knowledge about the decay constants and form factors entering the calculation. The pseudoscalar decay constants are (f_π, f_K, f_B). The values of the two former are taken from [4] while for the latter we use $f_B = 170\,\text{MeV}$ [22]. The second kind are the scalar decay constants (f_a, f_f) which are defined by (A.3) and (A.4) in the appendix and are related to $\tilde{f}_S$ through the relation $\tilde{f}_S = \frac{m_S f_S}{(m_1 - m_2)}$, where $m_{1,2}$ are the masses of the constituents quarks of the scalar S. In [23] theoretical arguments were used to estimate $\tilde{f}_{a_0, f_0}$. In Table 2, we quote the values we use for the scalar–pseudoscalar transition form factor $F_0^{SP}(m_B^2)$ which, as indicated, are evaluated at the m_B scale, i.e. can be calculated in the context of perturbation theory. The amplitude also involves form factors of the type $F_0^{BS}(m_P^2), F_0^{BP}(m_S^2)$, these are evaluated at the scale of the scalar mesons (around 1 GeV); therefore they cannot be computed using perturbative methods and few is known about their values. For the decay we are interested in ($B \to f_0 K^0, B^- \to f_0 K^-$ and $B \to a_0\pi$) we require $F_0^{B\pi}$, F_0^{BK}, $F_0^{Bf^0}$, $F_0^{Ba_0}$, $F_0^{a\pi}$ and $F_0^{f^0K}$. The two first ($F_0^{B\pi}, F_0^{BK}$) are relatively well known and we shall use the value reported in [24]. The remaining form factors can be determined using available experimental results[1] [3–5] given in Table 3, however since existing data involve large error bars, below we present an estimate of $F_0^{a_0\pi}$ based on the use of perturbative QCD. As explained in the following section, our interest in this form factor arises from the need to estimate the annihilation effects.

Table 2. Numerical values of the form factors

f_B	170 MeV	[22]
f_K	159.8 MeV	[2]
f_π	130.7 MeV	[2]
$\tilde{f}_{f_s^0}$	180 MeV	[23]
f_{a_0}	1 MeV	[23]
$\tilde{f}_{a_0}$	400 MeV	[23]
$F_0^{B^0\pi^-}$	0.28	[24]
$F_0^{B^0K^-}$	0.34	[24]

Table 3. Branching ratios of measured PS channel decays of B mesons

$\text{Br}(\bar{B}^0 \to \pi^\pm a_0^\mp)$	$(2.8^{+1.65}_{1.47})\ 10^{-6}$	[4]
$\text{Br}(\bar{B}^0 - \to \pi^- a_0^0)$	$(3.6^{+2.25}_{-2.06}\ 10^{-6}$	[4]
$\text{Br}(B^- \to K^- f^0)$	$(18.9.5^{+3.0}_{-2.8})\ 10^{-6}$	[3]
$\text{Br}(\bar{B}^0 \to K^0 f^0)$	$(13.3 \pm 3.6)\ 10^{-6}$	[3]

3 Annihilation form factors from perturbative QCD

At tree level $\bar{B}^0 \to \pi^+ a_0^-$ is strongly suppressed due to the absence of second class currents so that in order to calculate the associated branching ratio an estimate of the contribution of the B annihilation is necessary. The annihilation amplitude is proportional to $X^{\bar{B}^0}_{(a_0^- \pi^+)_u}$ which itself is proportional to $F_0^{a_0^- \pi^+}(m_B^2)$. Below we compute this form factor, assuming the scalar meson a_0^- is a two quark state and using the standard approach of perturbative QCD [25]. The whole approach is justified by the scale m_B^2 at which the form factor has to be evaluated.

In order to fix the conventions, we recall the form factor definition:

$$\langle M_2(p_2) | L_\mu | M_1(p_1) \rangle$$
$$= \left(p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2}\right)_\mu F_+^{M_2 M_1}(q^2)$$
$$+ \left(\frac{m_1^2 - m_2^2}{q^2}\right) q_\mu F_0^{M_1 M_2}(q^2), \tag{11}$$

[1] We used $\text{Br}(f^0 \to \pi^+\pi^-) \approx 0.45$ [2] in order to get the $\text{Br}(B^{-,0} \to K^{-,0} f^0)$ from published results.

with $q = p_1 - p_2$. Projecting the amplitude on q one obtains

$$q_\mu\langle M_2(p_2)|L^\mu|M_1(p_1)\rangle = (m_1^2 - m_2^2)F_0^{M_1,M_2}(q^2), \tag{12}$$

$$q_\mu\langle M_2(p_2)M_1(p_1)|L^\mu|0\rangle = (m_2^2 - m_1^2)F_0^{M_2,M_1}(q^2). \tag{13}$$

In the context of PQCD the contributions to both amplitudes have exactly the same structure, so following [26] we calculate $q_\mu\langle M_2(p_2)|L^\mu|M_1(p_1)\rangle$ and then $q_\mu\langle M_2(p_2)M_1(p_1)|L^\mu|0\rangle$ is obtained just by changing the sign of p_1. The form factors are expressed in terms of the distribution amplitudes:

$$\Psi_\pi(x,p) = \frac{-\mathrm{i}I_c}{\sqrt{2N_c}}\phi_\pi(x)(\hat{p} + m_\pi)\gamma_5, \tag{14}$$

$$\Psi_{a_0}(x,p) = \frac{I_c}{\sqrt{2N_c}}\phi_{a_0}(x)(\hat{p} + m_{a_0}), \tag{15}$$

where I_C is the identity in color space, $\hat{p} = \gamma_\mu p^\mu$ and

$$\int \phi_\pi(x)\mathrm{d}x = \frac{1}{2\sqrt{2N_c}}f_\pi, \tag{16}$$

$$\int \phi_{a_0}(x)\mathrm{d}x = \frac{1}{2\sqrt{2N_c}}f_{a_0}. \tag{17}$$

The wave functions $\phi_{\pi,a_0}(x)$ are given by [27]

$$\phi_\pi(x) = \frac{2N_c}{2\sqrt{2N_c}}f_\pi x(1-x) + \ldots, \tag{18}$$

$$\phi_{a_0}(x) = \frac{2N_c}{2\sqrt{2N_c}}f_{a_0}x(1-x)\left(1 + B_1C_1^{3/2}(2x-1)\right) + \ldots, \tag{19}$$

where $f_\pi = 130\,\mathrm{MeV}$, $|B_1 f_{a_0}| \simeq 75\,\mathrm{MeV}$, and $C_1^{3/2}(2x-1)$ is the Gegenbauer polynomial. Notice that we require only the combination $|B_1 f_{a_0}|$, so that we do not need to specify separately a value for f_{a_0}. Finally the matrix element is expressed as

$$\begin{aligned} &q_\mu\langle\pi(p_2)|L^\mu|a_0(p_1)\rangle \\ &= -C(R)\frac{T_r(I_C)}{2N_c}g_s^2\int \mathrm{d}x\mathrm{d}y\phi_{a_0}(x)\phi_\pi(y) \\ &\times\left\{\frac{\mathrm{Tr}\left[\gamma_5(\hat{p}_2 + m_\pi)\gamma_\nu\hat{P}_{1l}q_\mu L^\mu(\hat{p}_1 + m_{a_0})\gamma^\nu\right]}{k^2P_{1l}^2}\right. \\ &+\left.\frac{\mathrm{Tr}\left[\gamma_5(\hat{p}_2 + m_\pi)q_\mu L^\mu\hat{P}_{2l}\gamma^\nu(\hat{p}_1 + m_{a_0})\gamma_\nu\right]}{k^2P_{2l}^2}\right\}, \end{aligned} \tag{20}$$

where $C(R) = 4/3$, $(p_1 - p_2)^2 = q^2 = m_B^2$, $k = -xp_1 + (1-y)p_2$, $P_{1l} = k + yp_2$, $P_{2l} = -k + (1-x)p_1$ and

$$P_{1l}^2 = x^2m_{a_0}^2 + m_\pi^2 + x(m_B^2 - m_{a_0}^2 - m_\pi^2), \tag{21}$$

$$P_{2l}^2 = (1-y)m_B^2 + ym_{a_0}^2 - m_\pi^2 y(1-y). \tag{22}$$

Integrating numerically, one gets

$$\left|F_0^{a_0^-\pi^+}(m_B^2)\right| \approx 0.004. \tag{23}$$

It is important to remark that, since we only know $|B_1|$, the CP-conserving phase of the annihilation contributions is not fixed.

4 Numerical results for $B \to a\pi$ and $B \to f_0 K$

One can proceed along similar lines to describe other processes involving $a_0^{\pm,0}$ scalar mesons, but instead we use $SU(2)$ isospin symmetry and the quark content of the a_0 in order to obtain a relation between the form factors. As an alternative, in order to complement the information, we use the available experimental data to obtain constraints on the form factor values. It turns out that the consistency of the two sets of values so obtained provide further confidence on our approach.

We assume the conventional quark content of the pseudoscalar mesons [2] and parameterize the mixing in the scalar sector, in the strange–nonstrange basis, as follows:

$$\sigma = \cos\phi_S\bar{n}n - \sin\phi_S\bar{s}s, \tag{24}$$

$$f_0 = \sin\phi_S\bar{n}n + \cos\phi_S\bar{s}s, \tag{25}$$

where $\bar{n}n = (\bar{u}u + \bar{d}d)/\sqrt{2}$, and the singlet–octet mixing angle θ_S is related to ϕ_S by $\phi_S - \theta_S = \cos^{-1}[1/\sqrt{3}] \simeq 55°$. A diagrammatic analysis of the contributions to the form factor based upon the quark composition and $SU(2)$ isospin symmetry between the up and down quarks leads to the following relations:

$$\frac{\sqrt{2}}{\cos\phi_S}F_0^{B^-\sigma} = \frac{\sqrt{2}}{\sin\phi_S}F_0^{B^-f_0} = \frac{\sqrt{2}}{\sin\phi_S}F_0^{\bar{B}^0f_0} = F_0^{\bar{B}^0a_0^+}. \tag{26}$$

From these relations it follows that $|F_0^{\bar{B}^0f_0}| < |F_0^{\bar{B}^0a_0^+}|/\sqrt{2}$.

Similar relations between the annihilation form factors ($F_0^{a_0\pi}$ and $F_0^{f^0K}$) could be obtained in terms of the $SU(3)$ symmetry, however we will not follow this approach since large deviations from the symmetry limit are expected. We determine the allowed values of the form factors $F_0^{a_0\pi}$ ($F_0^{f_0K}$) and $F_0^{Ba_0}$ (F_0^{BK}) using the experimental results given in Table 3, the scalar meson masses reported in [2] and the numerical values given in Table 2. The results are summarized in Figs. 1 and 2. Assuming that perturbative QCD leads us to the right order of magnitude for $F_0^{a_0\pi}$, it follows that

$$0.14 \leq |F_0^{Ba_0}| \leq 0.21. \tag{27}$$

This result is compatible (even if slightly smaller) with the model-dependent predictions $F_0^{\bar{B}^0a_0^+}(0) = 0.55 \pm 0.22$ [28]. Using (26), it follows that

$$|F_0^{\bar{B}^0f_0}| \leq 0.20. \tag{28}$$

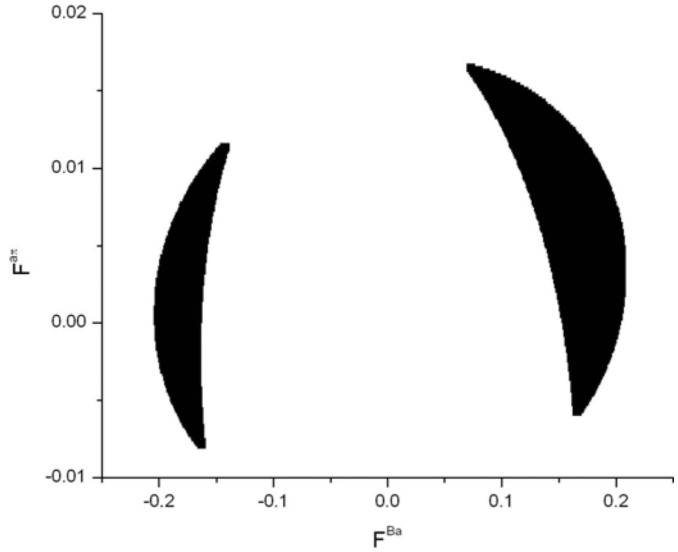

Fig. 1. Allowed region for $F_0^{B^0a^+}$ and $F_0^{\pi a}$ at one σ using experimental data on $\mathrm{Br}(B^0 \to \pi^- a_0^+)$ and $\mathrm{Br}(B^- \to \pi^- a_0^0)$

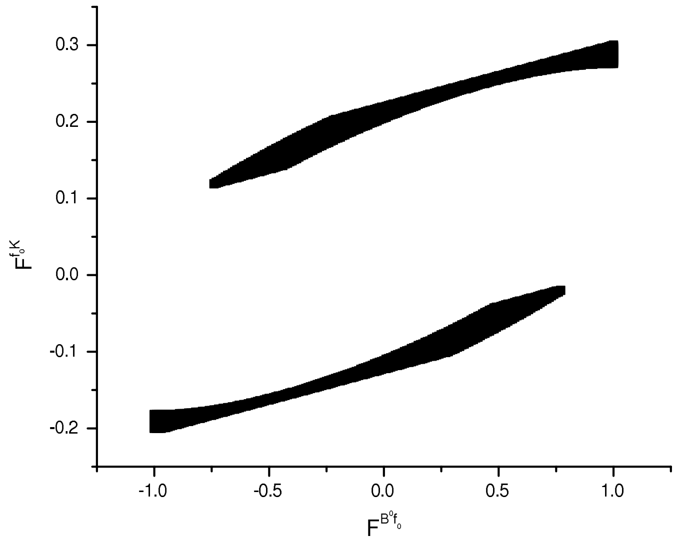

Fig. 2. Allowed region for parameters $F_0^{B_0 f_0}$ and $F_0^{K^0 f_0}$ at one σ

Figure 2 shows the $F_0^{\bar{B}^0 f_0}$–$F^{f_0 K}$ plane, the dark regions corresponding to values compatible with the experimental data when one standard deviation is allowed. One observes that $|F_0^{\bar{B}^0 f_0}| \le 0.20$ requires a large $|F_0^{f_0 K}|$ value, in fact the smallest value for $|F_0^{f_0 K}|$ is around 0.05, which is more than one order of magnitude bigger than the PQCD prediction for $|F_0^{a_0\pi}|$. In this respect it is interesting to note that one could argue that

$$\frac{|F_0^{f_0K}|}{|F_0^{a_0\pi}|} \approx \frac{m_K^2}{m_\pi^2} \approx 12. \tag{29}$$

Once the values of the form factors $F_0^{Bf_0}$ and $F_0^{Ba_0}$ have been constrained, we turn our attention to the subdominant processes $\bar{B}^0 \to \pi^+ a_0^-$ which is strongly suppressed by G parity and isospin. Using the estimate for the annihilation contribution presented above, a prediction for the branching ratio $\mathrm{Br}(\bar{B}^0 \to \pi^+ a_0^-)$ follows from (5). Varying between 0 and $\boldsymbol{\pi}$ the CP-conserving phase for the annihilation, it follows that

$$10^{-9} \le \mathrm{Br}(\bar{B}^0 \to \pi^+ a_0^-) \le 4 \times 10^{-7}, \tag{30}$$

where the lower limit is obtained when annihilation and the remaining contributions interfere destructively while the upper limit applies when the interference is constructive.

Another channel suppressed by G parity is $B^- \to \pi^0 a_0^-$, for which we obtain

$$6.4 \times 10^{-8} \le \mathrm{Br}(B^- \to \pi^0 a_0^-) \le 2.4 \times 10^{-7}. \tag{31}$$

In the following we include a few comments regarding the four quark structure of the scalar mesons. In [6] the author concludes that the positive identification of $B^0/\bar{B}^0 \to a_0^\pm \pi^\mp$ is evidence against the four quark assignment of a_0, or else for the breakdown of perturbative QCD. Several models where the scalars are four quark states [12, 29–31] have been proposed but at present time no model is favored . In order to make a statement below we apply our approach to a particular model and following the authors in [29] we assume that the quark content of the scalars is

$$\begin{aligned}
a_0^+ &= ud\bar{d}\bar{s}, \quad a_0^- = ds\bar{u}\bar{s}, \quad a_0^0 = \frac{1}{\sqrt{2}}\left(us\bar{u}\bar{s} - ds\bar{d}\bar{s}\right), \\
K_0^+ &= ud\bar{d}\bar{s}, \quad K_0^0 = ud\bar{u}\bar{s}, \quad \bar{K}_0^0 = us\bar{u}\bar{d}, \\
K_0^- &= ds\bar{u}\bar{d}, \\
f_0 &= \frac{\cos\phi}{\sqrt{2}}\left(su\bar{s}\bar{u} + sd\bar{s}\bar{d}\right) + \sin\phi\ ud\bar{u}\bar{d}, \\
\sigma &= -\frac{\sin\phi}{\sqrt{2}}\left(su\bar{s}\bar{u} + sd\bar{s}\bar{d}\right) + \cos\phi\ ud\bar{u}\bar{d},
\end{aligned} \tag{32}$$

where the mixing angle is obtained from the relation $\tan\phi = -0.19$ (for $m_\sigma = 0.47\,\mathrm{GeV}$), so $\phi = -5.4°$ and $84.6°$.

It is well known that perturbative QCD predicts that the form factor behaves like $1/q^{2(n-1)}$ where n is the number of constituents of the hadron. If $n = 4$, $F_0^{a_0^-\pi^+}(m_B^2)$ is strongly suppressed and annihilation can be neglected ($F_0^{a_0\pi} = F_0^{Kf_0} = 0$). From Figs. 2 and 1, and using the fact that in four quark models for scalars annihilation can safely be neglected, one concludes that

$$0.70 \le\ F_0^{Bf_0}\ \le 0.75, \tag{33}$$

$$0.15 \le |F_0^{Ba_0}| \le 0.20. \tag{34}$$

Note that the $|F_0^{Ba_0}|$ value is close to (27), obtained assuming that the scalars are two quark states. These results are presented in Fig. 3.

On the other hand, using (5), the branching ratio for the $\bar{B}^0 \to \pi^+ a_0^-$ is calculated. Assuming the four quark model for the a_0, where the annihilation is strongly suppressed ($F_0^{a_0\pi} = F_0^{Kf_0} = 0$) one obtains $\mathrm{Br}(\bar{B}^0 \to \pi^+ a_0^-) \approx 10^{-7}$. Thus, comparing with (30), our results imply that the B

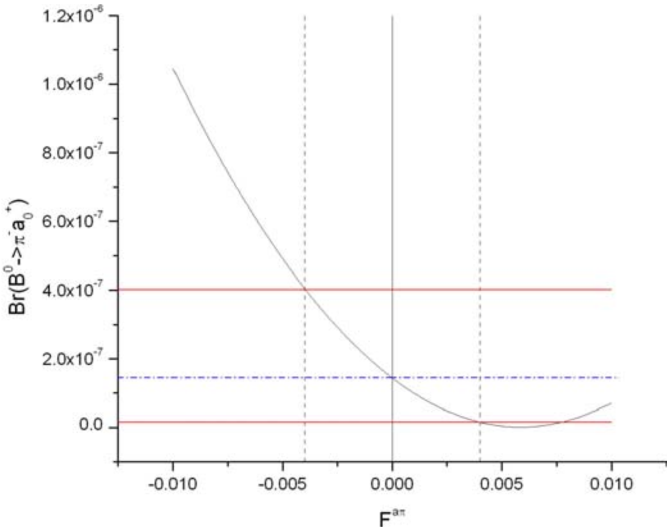

Fig. 3. Branching ratio for $\mathrm{Br}(\bar{B}^0 \to \pi^+ a_0^-)$. The horizontal dot-dashed line correspond to no annihilation contribution. The band between the two horizontal continuous lines is obtained by varying $F^{a\pi}$ phase between 0 and $typesetter, this Greek lowercase pi roman, please : \pi$

decays so far considered cannot be used to distinguish between the two and four quark assignment of the a_0, unless one can fix the annihilation phase so as to avoid the ambiguity of the interference terms. Indeed the interference terms can be large enough so that the predictions of the two models are incompatible. We conclude that the positive identification of $\bar{B}^0 \to \pi^+ a_0^-$ should not be considered as evidence against the four quark assignment of $a_0(980)$. This is in contrast with [6], where the annihilation contribution is not quantified. Relevant for this conclusion is the ambiguity in the CP-conserving phase of the annihilation contributions. To finish this work it is interesting to remark that a better channel exists to distinguish between the two and four quark models for $a_0(980)$. Indeed, for four quark models of the $a_0(980)$, with the value for $|F_0^{Ba_0}|$ previously determined, one obtains

$$2 \times 10^{-9} \leq \mathrm{Br}(B^- \to \pi^0 a_0^-) \leq 10^{-8}; \qquad (35)$$

this is to be compared with the value obtained in the model with scalars as two quark states:

$$6.4 \times 10^{-8} \leq \mathrm{Br}(B^- \to \pi^0 a_0^-) \leq 2.4 \times 10^{-7}. \qquad (36)$$

Thus predictions of models where the scalars are four quark states are typically one order of magnitude smaller than those where the scalars are two quark states.

5 Conclusions

In this paper we considered processes for which the leading contribution is suppressed. Using the factorization approximation and available experimental data we estimated the effect of the annihilation contribution to the processes $\bar{B}^0 \to \pi^\pm a_0^\mp$ and $\bar{B}^{0,-} \to K^{0,-} f_0$. We have shown that a consistent picture can be obtained, although important contributions from annihilation penguins to $\bar{B}^{0,-} \to K^{0,-} f_0$ are required.

We evaluated the $F^{a_0\pi}$ form factor using the perturbative QCD approach, which was used to estimate the annihilation contribution to suppressed processes like $B^- \to \pi^0 a_0^-$ and $\bar{B}^0 \to \pi^+ a_0^-$.

We calculated contributions neglected in [6], and also qualified the statement "*the positive identification of* $B^0/\bar{B}^0 \to a_0^\pm \pi^\mp$ *is evidence against the four quark assignment of* a_0 *or else, for the breakdown of perturbative QCD*". According to our results the B decays considered in [6] cannot be used to distinguish between the two and four quark assignment of the a_0, unless an independent determination of the annihilation phase can be obtained. For the decay $B^- \to \pi^0 a_0^-$ the two and four quark models predict branching ratios that differ in one order of magnitude.

Acknowledgements. The work of D.D. was supported by project PROMEP/103.5/04/1335 (México) and the work of J.L.M. was supported by CONACyT, México under project 37234-E and project CONACyT SEP-2003-C02-44644. C.A.R. wants to thank the hospitality of J.L.M. and D.D. at IFUG, where part of this work was done.

Appendix A: $X^a_{b,c}$ expressed in terms of the form factors

Below we list the $X^a_{b,c}$ expressed in terms of the form factors. We quote only those needed to compute the branching ratios given in the paper:

$$\begin{aligned}
X^{\pi^-}_{\bar{B}^0 a_0^+} &= \langle \pi^- |(\bar{d}u)_{\mathrm{L}}|0\rangle \langle a_0^+|(\bar{u}b)_{\mathrm{L}}|\bar{B}^0\rangle \\
&= f_\pi (m_B^2 - m_a^2) F_0^{\bar{B}^0 a_0^+}(m_\pi^2), \\
X^{a_0^-}_{\bar{B}^0 \pi^+} &= \langle a_0^- |(\bar{d}u)_{\mathrm{L}}|0\rangle \langle \pi^+|(\bar{u}b)_{\mathrm{L}}|\bar{B}^0\rangle \\
&= -f_a (m_B^2 - m_\pi^2) F_0^{\bar{B}^0 \pi^+}(m_a^2), \\
X^{a_0^-}_{B^- \pi^0} &= \langle a_0^- |(\bar{d}u)_{\mathrm{L}}|0\rangle \langle \pi^0|(\bar{u}b)_{\mathrm{L}}|B^-\rangle \\
&= f_a (m_B^2 - m_\pi^2) F_0^{B^- \pi^0}(m_a^2), \\
X^{\pi_u^0}_{B^- a_0^-} &= \langle \pi^0 |(\bar{u}u)_{\mathrm{L}}|0\rangle \langle a_0^-|(\bar{d}b)_{\mathrm{L}}|B^-\rangle \\
&= \frac{f_\pi}{\sqrt{2}} (m_B^2 - m_a^2) F_0^{B^- \pi^-}(m_\pi^2), \\
X^{\pi^-}_{B^- S^0} &= \langle \pi^- |(\bar{u}d)_{\mathrm{L}}|0\rangle \langle S^0|(\bar{u}b)_{\mathrm{L}}|B^-\rangle \\
&= f_\pi (m_B^2 - m_{S^0}^2) F_0^{B^- S^0}(m_\pi^2), \\
X^{B^-}_{S^0 \pi^-} &= \langle S^0 P^- |(\bar{d}u)_{\mathrm{L}}|0\rangle \langle 0|(\bar{u}b)_{\mathrm{L}}|B^-\rangle \\
&= -f_B (m_{S_0}^2 - m_\pi^2) F_0^{S_0 \pi^-}(m_B^2), \\
X^{B^-}_{a_0^- \pi^0} &= \langle a_0^- \pi^0 |(\bar{d}u)_{\mathrm{L}}|0\rangle \langle 0|(\bar{u}b)_{\mathrm{L}}|B^-\rangle
\end{aligned}$$

$$
\begin{aligned}
&= -f_B(m_a^2 - m_\pi^2) F_0^{a_0^- \pi^0}(m_B^2), \\
X_{\bar{K}^0 S_0}^{\bar{B}^0} &= \langle \bar{K}^0 S^0 | (\bar{s}d)_{\mathrm{L}} | 0 \rangle \langle 0 | (\bar{d}b)_{\mathrm{L}} | \bar{B}^0 \rangle \\
&= -f_B(m_{S^0}^2 - m_K^2) F_0^{S^0 \bar{K}^0}(m_B^2), \\
X_{S_0 K^-}^{B^-} &= \langle S^0 K^- | (\bar{s}u)_{\mathrm{L}} | 0 \rangle \langle 0 | (\bar{u}b)_{\mathrm{L}} | B^- \rangle \\
&= -f_B(m_S^2 - m_K^2) F_0^{S^0 K^-}(m_B^2), \\
X_{(a_0^- \pi^+)_u}^{\bar{B}^0} &= \langle a_0^- \pi^+ | (\bar{u}u)_{\mathrm{L}} | 0 \rangle \langle 0 | (\bar{d}b)_{\mathrm{L}} | \bar{B}^0 \rangle \\
&= -f_B(m_{a_0^-}^2 - m_\pi^2) F_0^{a_0^- \pi^+}(m_B^2), \\
X_{a_0^+ \pi^-}^{\bar{B}^0} &= \langle a_0^+ \pi^- | (\bar{u}u)_{\mathrm{L}} | 0 \rangle \langle 0 | (\bar{d}b)_{\mathrm{L}} | \bar{B}^0 \rangle \\
&= -f_B(m_{a_0^-}^2 - m_\pi^2) F_0^{a_0^+ \pi^-}(m_B^2), \\
X_{\bar{B}^0 S^0}^{\bar{K}^0} &= \langle \bar{K}^0 | (\bar{s}d)_{\mathrm{L}} | 0 \rangle \langle S^0 | (\bar{d}b)_{\mathrm{L}} | \bar{B}^0 \rangle \\
&= f_{\bar{K}^0}(m_B^2 - m_{S^0}^2) F_0^{\bar{B}^0 S^0}(m_K^2), \\
X_{B^- S^0}^{K^-} &= \langle K^- | (\bar{u}s)_{\mathrm{L}} | 0 \rangle \langle S^0 | (\bar{u}b)_{\mathrm{L}} | B^- \rangle \\
&= f_K(m_B^2 - m_{S^0}^2) F_0^{B^- S^0}(m_\pi^2), \\
\tilde{X}_{\bar{B}^0 \pi^+}^{a_0^-} &= \langle a_0^- | \bar{d}u | 0 \rangle \langle \pi^+ | \bar{u}b | \bar{B}^0 \rangle \\
&= m_a \tilde{f}_{a_0^-} \frac{m_B^2 - m_\pi^2}{m_b - m_u} F_0^{\bar{B}^0 \pi^+}(m_a^2), \\
\tilde{X}_{B^- \pi^-}^{S_d^0} &= \langle S^0 | \bar{d}d | 0 \rangle \langle \pi^- | \bar{d}b | B^- \rangle \\
&= \frac{m_{S_0} \tilde{f}_{S_d^0}}{m_b - m_d} (m_B^2 - m_\pi^2) F_0^{B^- \pi^-}(m_{S_0}^2), \\
\tilde{X}_{B^- \pi^0}^{a_0^-} &= \langle a_0^- | \bar{d}u | 0 \rangle \langle \pi^0 | \bar{u}b | B^- \rangle \\
&= m_a \tilde{f}_{a_0^-} \frac{m_B^2 - m_\pi^2}{m_b - m_d} F_0^{B^- \pi^0}(m_a^2), \\
\tilde{X}_{B^- K^-}^{S_s^0} &= \langle S^0 | \bar{s}s | 0 \rangle \langle K^- | \bar{s}b | B^- \rangle \\
&= m_{S^0} \tilde{f}_{S_s^0} \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{B^- K^-}(m_{S^0}^2), \\
\tilde{X}_{\bar{B}^0 \bar{K}^0}^{S_s^0} &= \langle S^0 | \bar{s}s | 0 \rangle \langle \bar{K}^0 | \bar{s}b | \bar{B}^0 \rangle \\
&= m_{S^0} \tilde{f}_{S_s^0} \frac{m_B^2 - m_K^2}{m_b - m_s} F_0^{\bar{B}^0 \bar{K}^0}(m_{S^0}^2) \\
&= \tilde{X}_{B^- K^-}^{S_s^0},
\end{aligned} \tag{A.1}
$$

where S_0 is a neutral scalar (a_0^0 or f^0).

Appendix B: Form factors definitions and conventions

In order to compute the amplitude using factorization, we use the following parametrization of the form factors. The decay constants are defined by

$$\langle 0 | A_\mu | P(q) \rangle = \mathrm{i} f_P q_\mu, \tag{A.1}$$

$$\langle 0 | \bar{q}_1 \gamma_5 q_2 | P(q) \rangle \simeq \frac{-\mathrm{i} f_P m_P^2}{m_1 + m_2} \equiv \bar{f}_P m_P, \tag{A.2}$$

$$\langle a_0^- | \bar{d} \gamma_\mu u | 0 \rangle = f_{a0} p_\mu, \tag{A.3}$$

$$\langle a_0^- | \bar{d}u | 0 \rangle = m_{a0} \bar{f}_{a0}. \tag{A.4}$$

Using the equations of motion $(-\mathrm{i}\partial^\mu(\bar{q}_1 \gamma_\mu \gamma_5 q_2) = (m_1 + m_2) \bar{q}_1 \gamma_5 q_2$ and $-\mathrm{i}\partial^\mu(\bar{q}_1 \gamma_\mu q_2) = (m_1 - m_2) \bar{q}_1 q_2$ [17,19] one can show that $\bar{f}_S = m_S f_S/(m_1 - m_2)$ and that $f_{S^0} = 0$ for a neutral scalar. Form factors are defined by

$$
\begin{aligned}
&\langle M_2(p_2) | L_\mu | M_1(p_1) \rangle \\
&= \left(p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2} q \right)_\mu F_+^{M_1 M_2} \\
&\quad + \frac{m_1^2 - m_2^2}{q^2} q_\mu F_0^{M_1 M_2}(q^2),
\end{aligned} \tag{A.5}
$$

$$
\begin{aligned}
&\langle M_2(p_2) M_1(p_1) | L_\mu | 0 \rangle \\
&= \left(p_2 - p_1 - \frac{m_2^2 - m_1^2}{q^2} q \right)_\mu F_+^{M_2 M_1}(q^2) \\
&\quad + \frac{m_2^2 - m_1^2}{q^2} q_\mu F_0^{M_2 M_1}(q^2),
\end{aligned} \tag{A.6}
$$

where $L_\mu = \gamma^\mu \frac{1-\gamma_5}{2} \gamma^\mu P_{\mathrm{L}}$. A factor of $-\mathrm{i}$ has to be added to the form factors in case one of the mesons is scalar.

References

1. P. Ball et al., hep-ph/0003238, http://lhcb.web.cern.ch/lhcb; K. Anikeev et al., hep-ph/0201071, http://www-btev.fnal.gov; http//www.hera-b.desy.de; A.G. Akeroyd et al., hep-ex/0406071; I.I. Bigi, A.I. Sanda, hep-ph/0401003
2. S. Eidelman et al. [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004) pdg.lbl.gov
3. B. Aubert et al. [BABAR Collaboration], hep-ex/0408095; hep-ex/0406040; A. Garmash et al. [Belle Collaboration], Phys. Rev. D **69**, 012001 (2004) [hep-ex/0307082]; S. Laplace [BABAR Collaboration], hep-ex/0309049; K. Abe et al. [Belle Collaboration], Phys. Rev. D **65**, 092005 (2002) [hep-ex/0201007]
4. B. Aubert et al. [BABAR Collaboration], hep-ex/0407013; J.G. Smith, hep-ex/0406063; B. Aubert et al. [BABAR Collaboration], hep-ex/0107075
5. S. Laplace, V. Shelkov, Eur. Phys. J. C **22**, 431 (2001) (hep-ph/0105252)
6. M. Suzuki, Phys. Rev. D **65**, 097501 (2002)
7. See for example, S. Spanier, N.A. Tornqvist, Phys. Lett. B **592**, 1 (2004); C. Amsler, N.A. Tornqvist Phys. Rep. **389**, 61 (2004) and references therein
8. F.E. Close, N.A. Tornqvist, J. Phys. G **28**, R249 (2002) [hep-ph/0204205]; A. Abdel-Rehim, D. Black, A.H. Fariborz, J. Schechter, AIP Conf. Proc. **687**, 51 (2003); E. van Beveren, G. Rupp, Mod. Phys. Lett. A **19**, 1949

(2004); M. Uehara hep-ph/0404221; M.R. Pennington, eConf C**030614**, 002 (2003) [hep-ph/0310186]; J.L. Lucio Martinez, M. Napsuciale, Phys. Lett. B **454**, 365 (1999) [hep-ph/9903234]; J.L. Lucio Martinez, J. Pestieau, Phys. Rev. D **42**, 3253 (1990); N.N. Achasov, V.N. Ivanchenko, Nucl. Phys. B **315**, 465 (1989); F.E. Close, N. Isgur, S. Kumano, Nucl. Phys. B **389**, 513 (1993); V.V. Anisovich, A.V. Sarantsev, Eur. Phys. J. A **16**, 229 (2003) [hep-ph/0204328]; F. De Fazio, M.R. Pennington, Phys. Lett. B **521**, 15 (2001) [hep-ph/0104289]

9. R. Akhmetshin et al. [CMD-2 Collaboration], Phys. Lett. B **462**, 380 (1999) [hep-ex/9907006]; V.M. Aulchenko et al. [SND Collaboration], Phys. Lett. B **440**, 442 (1998) [hep-ex/9807016]; M.N. Achasov et al., Phys. Lett. B **485**, 349 (2000) [hep-ex/0005017]; J.E. Augustin et al. [DM2 Collaboration], Nucl. Phys. B **320**, 1 (1989)
10. E.M. Aitala et al. [E791 Collaboration], Phys. Rev. Lett. **89**, 121801 (2002) [hep-ex/0204018]; M. Ishida, S. Ishida, T. Komada, S.I. Matsumoto, Phys. Lett. B **518**, 47 (2001); H. Muramatsu et al. [CLEO Collaboration], Phys. Rev. Lett. **89**, 251802 (2002) [Erratum **90**, 059901 (2003)] [hep-ex/0207067]
11. D.V. Bugg, Phys. Lett. B **572**, 1 (2003) [Erratum B **595**, 556 (2004)]; G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B **603**, 125 (2001) [hep-ph/0103088]; R. Escribano, A. Gallegos, J.L. Lucio M, G. Moreno, J. Pestieau, Eur. Phys. J. C **28**, 107 (2003) [hep-ph/0204338]; A. Gallegos, J.L. Lucio M., J. Pestieau, Phys. Rev. D **69**, 074033 (2004) [hep-ph/0311133]
12. J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D **59**, 074001 (1999) [Erratum D **60**, 099906 (1999)] [hep-ph/9804209]; J.A. Oller, Nucl. Phys. A **727**, 353 (2003) [hep-ph/0306031]
13. V. Chernyak, Phys. Lett. B **509**, 273 (2001) [hep-ph/0102217]; A.C. Katoch, R.C. Verma, Int. J. Mod. Phys. A **11**, 129 (1996)
14. See for example, T.V. Brito, F.S. Navarra, M. Nielsen, M.E. Bracco, hep-ph/0411233; A. Gokalp, Y. Sarac, O. Yilmaz, hep-ph/0410380; H.G. Dosch, E.M. Ferreira, F.S. Navarra, M. Nielsen, Phys. Rev. D **65**, 114002 (2002) [hep-ph/0203225]; A. Ali, V.M. Braun, H. Simma, Z. Phys. C **63**, 437 (1994) [hep-ph/9401277]; V.M. Braun, A.V. Kolesnichenko, Phys. Lett. B **175**, 485 (1986) [Sov. J. Nucl. Phys. **44**, 756 (1986)
15. P. Minkowski, W. Ochs, hep-ph/0404194; hep-ph/0304144; Nucl. Phys. Proc. Suppl. **121**, 123, (2003) [hep-ph/0209225]; Nucl. Phys. Proc. Suppl. **121**, 119 (2003) [hep-ph/0209223]; Eur. Phys. J. C **9**, 283 (1999) [hep-ph/9811518]; E. Klempt, B.C. Metsch, C.R. Munz, H.R. Petry, Phys. Lett. B **361**, 160 (1995) [hep-ph/9507449]; E. Klempt, hep-ex/0101031; V.V. Anisovich, A.V. Sarantsev, Eur. Phys. J. A **16**, 229 (2003) [hep-ph/0204328]
16. C.H. Chen, Phys. Rev. D **67**, 094011 (2003) [hep-ph/0302059]; D **67**, 014012 (2003) [hep-ph/0210028]; H.Y. Cheng, Phys. Rev. D **67**, 034024 (2003) [hep-ph/0212117]
17. A. Ali, G. Kramer, C.D. Lu, Phys. Rev. D **58**, 094009 (1998) [hep-ph/9804363]; A. Ali, C. Greub, Phys. Rev. D **57**, 2996 (1998) [hep-ph/9707251]
18. M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B **606**, 245 (2001) [hep-ph/0104110]; G. Buchalla, A.J. Buras, M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [hep-ph/9512380]
19. Y.H. Chen, H.Y. Cheng, B. Tseng, K.C. Yang, Phys. Rev. D **60**, 094014 (1999) [hep-ph/9903453]
20. Y.Y. Keum, H. n. Li, Phys. Rev. D **63**, 074006 (2001) [hep-ph/0006001]; Y.Y. Keum, H.N. Li, A.I. Sanda, Phys. Rev. D **63**, 054008 (2001) [hep-ph/0004173]; Y.Y. Keum, A.I. Sanda, Phys. Rev. D **67**, 054009 (2003) [hep-ph/0209014]; Y.Y. Keum, H. n. Li, A.I. Sanda, Phys. Lett. B **504**, 6 (2001) [hep-ph/0004004]
21. C.D. Lu, K. Ukai, M.Z. Yang, Phys. Rev. D **63**, 074009 (2001) [hep-ph/0004213]; C.D. Lu, M.Z. Yang, Eur. Phys. J. C **23**, 275 (2002) [hep-ph/0011238]; C.D. Lu, D.X. Zhang, Phys. Lett. B **400**, 188 (1997) [hep-ph/9703439]
22. M. Guagnelli, F. Palombi, R. Petronzio, N. Tantalo, Nucl. Phys. Proc. Suppl. **119**, 616 (2003) [hep-lat/0209113]; D. Becirevic, hep-ph/0310072
23. K. Maltman, Phys. Lett. B **462**, 14 (1999) [hep-ph/9906267]; Y. Meurice, Phys. Rev. D **36**, 2780 (1987); Mod. Phys. Lett. A **2**, 699 (1987); C.M. Shakin, H. Wang, Phys. Rev. D **63**, 074017 (2001); F. De Fazio, M.R. Pennington, Phys. Lett. B **521**, 15 (2001) [hep-ph/0104289]
24. A. Abada, D. Becirevic, P. Boucaud, J.P. Leroy, V. Lubicz, F. Mescia, Nucl. Phys. B **619**, 565 (2001) [hep-lat/0011065]; D. Becirevic, S. Prelovsek, J. Zupan, Phys. Rev. D **68**, 074003 (2003) [hep-lat/0305001]; D. Bećrević, S. Prelovšek, Ju. Zupan, Phys. Rev. D **67**, 054010 (2003); ICHEP-02, Amsterdam 2002; P. Ball, R. Zwicky, hep-ph/0406261
25. G.P. Lepage, S.J. Brodsky, Phys. Rev. D **22**, 2157 (1980); G.R. Farrar, D.R. Jackson, Phys. Rev. Lett. **43**, 246 (1979); S. Brodsky, Lett. Nuov. Cim. **7**, 719 (1973); S.J. Brodsky, G.R. Farrar, Phys. Rev. D **11**, 1309 (1975); Phys. Rev. Lett. **31**, 1153 (1973)
26. D. s. Du, D. Yang, G. Zhu, Phys. Rev. D **60**, 054015 (1999)
27. M. Diehl, G. Hiller, JHEP **0106**, 067 (2001) [hep-ph/0105194]
28. A. Deandrea, A.D. Polosa, Phys. Rev. Lett. **86**, 216 (2001) [hep-ph/0008084]
N. Paver, Riazuddin, hep-ph/0107330
29. L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, hep-ph/0407017
30. D. Black, A.H. Fariborz, F. Sannino, J. Schechter, Phys. Rev. D **59**, 074026 (1999) [hep-ph/9808415]
31. M. Napsuciale, S. Rodriguez, Int. J. Mod. Phys. A **16**, 3011 (2001) [hep-ph/0204149]